Multi-population Firefly Algorithm

Jani Dugonik
Faculty of Electrical Engineering and Computer Science
University of Maribor
2000 Maribor, Slovenia
jani.dugonik@um.si

Iztok Fister
Faculty of Electrical Engineering and Computer Science
University of Maribor
2000 Maribor, Slovenia
iztok.fister@um.si

ABSTRACT

This paper proposes a meta-heuristic Multi-Population Firefly Algorithm (MPFA) for single-modal optimization using two multi-population models, i.e., one is based on the island model while the other on the mainland-island model. The unique characteristics of each sub-population is evolved independently and the diversity of the entire population is effectively increased. Sub-populations communicate with each other to exchange information in order to expand the search range of the entire population. In line with this, each sub-population explores a specific part of the search space and contributes its part for exploring the global search space. The main goal of this paper was to analyze the performance between MPFA and the original Firefly Algorithm (FA). Experiments were performed on a CEC 2014 benchmark suite consisting of 16 single-objective functions and the obtained results show improvements in most of them.

Keywords

swarm intelligence, island model, multi-population, firefly algorithm

1. INTRODUCTION

An optimization problem is defined as a quadruple $OP = \langle I, S, f, goal \rangle$, where I denotes a set of all input instances $\mathbf{x} \in I$ in the form of $\mathbf{x} = \{x_1, x_2, ..., x_D\}$ where D is the dimensionality of the problem, $S(x)$ a set of all feasible solutions $\mathbf{y} = S(\mathbf{x})$, f is the objective function estimating the feasible solution, and *goal* determines an optimal criteria that can be either the minimum or maximum value of the objective function. A task of the optimization algorithm is to find the value of $\mathbf{y}^*$ that minimizes or maximizes (depending on the *goal*) the value of the objective function $f(\mathbf{y})$. The domain values of input variables $x_i \in [lb_i, ub_i]$ are limited by their lower lb_i and upper ub_i bounds.

Nature has evolved over millions of years and has found perfect solutions to almost all encountered problems. We can thus learn the success of problem-solving from nature and develop nature-inspired heuristic and/or meta-heuristic algorithms in order to solve optimization problems with which developers are confronted today. Two sources from the nature have particularly inspired developers of new optimization algorithms, i.e., Darwinian evolution and the behavior of social living insects (e.g., ants, bees, termites, etc.) and other creatures (e.g., birds, dolphins, fireflies, etc.). As a result, two main classes of nature-inspired algorithms exist nowadays, i.e., evolutionary algorithms (EA) [3] and swarm intelligence (SI)-based algorithms [1]. While the former already came in mature years, the latter has experienced rapid development. Almost every day, we are witnessing the birth of a new SI-based algorithm.

One of the younger members of the SI-based algorithms is the Firefly Algorithm (FA) as proposed by Yang in [11]. FA is inspired by a chemical phenomenon bioluminiscence needed by natural fireflies to find their prey, on the one hand, and to attract their mating partners, on the other hand. This algorithm belongs to a class of population-based algorithms [5, 4, 11, 12]. Population-based approaches maintain and improve multiple candidate solutions, often using population characteristics to guide the search. Two major components of any population-based search algorithms are exploitation and exploration. Exploitation refers to searching within neighborhoods for the best solutions and ensures that the solutions can converge into optimality, while the exploration uses randomization in order to avoid the solutions being trapped within a local optima and while at the same time increasing the diversity of the solutions. A good combination of these two components may usually ensure that the global optimality is achieved [11].

One of the possible ways of how to improve the exploration and exploitation in the original FA algorithm can be splitting the FA population into more sub-populations (so-called multi-populations). For instance, the authors in [10] presented a multi-population FA for correlated data routing within underwater wireless sensor networks. They designed three kinds of fireflies and their coordination rules in order to improve the adaptabilities of building, selecting, and optimizing a routing path. Groups are represented as sub-populations, where each sub-population conducts its own optimization in order to improve the convergence speed and solution precision of the algorithm. The author in [13] analyzed the ability of a multi-population differential evolution to locate all optima of a multi-modal function. The exploration was ensured by the controled initialization of sub-populations while a particular differential evolution algorithm ensured the exploitation. Sub-populations were communicating via archive where all located optima were stored. The authors in [8] used an evolutionary algorithm on punctuated equilibria. The theory of punctuated equilibria calls for the population to be split into several sub-populations. These sub-populations have isolated evolution (computation) and scattered with migration (communication).

This paper aimed to evaluate whether it is possible to outperform the performance of the original FA algorithm by splitting its population into more sub-populations. The proposed multi-population FA (MPFA) supports two multi-population models, i.e., Island [13] and Mainland-Island [9]. In these multi-population models, sub-populations evolve independently, thus the unique characteristics of each sub-population can be effectively maintained, and the diversity of the entire population is effectively increased. Sub-populations communicate with each other by exchanging information in order to expand the search range of the entire population. The search technique based on a population has proved to have good ability regarding global searching and can find a set of solutions in one-shot operation. The proposed multi-population FAs were compared with the original FA on single-objective CEC 2014 benchmark functions [7].

The remainder of this paper is organized as follows. In Section 2 the original FA will be presented. In Section 3 a multi-population FA with two multi-population models are presented in detail. Section 4 presents experiments, where the number of tests were performed in order to compare the proposed approach with the original FA. The paper is concluded with Section 5, where our opinion on the obtained results is given.

2. THE ORIGINAL FIREFLY ALGORITHM

The Firefly Algorithm (FA) [11] has two fundamental factors: light intensity and attractiveness. Light intensity I reflects the firefly location and determines its direction of movement, while the degree of attractiveness determines the distance that a firefly has moved. Both factors are constantly updated in order to achieve the objective of the optimization.

For simplicity, the author in [11] used the following three idealized rules:

- All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.
- Attractiveness is proportional to their brightness and for any two flashing fireflies, the dimmer one will move towards the brighter one. They both decrease as their distance increases. If there is no brighter one, it will move randomly.
- The brightness of a firefly is affected or determined by the landscape of the objective function.

Based on these three rules, the basic steps of the firefly algorithm (FA) can be summarized as the pseudo-code shown in Algorithm 1. Light intensity of a firefly is defined as:

Algorithm 1 Firefly Algorithm

Objective function $f(\mathbf{x})$, $\mathbf{x} = (x_1, ..., x_D)^T$
Generate initial population of fireflies $\mathbf{x}_i$ (i=1,2,...,Np)
Light intensity I_i at $\mathbf{x}_i$ is determined by $f(\mathbf{x}_i)$
Define light absorption coefficient γ
while $(t < G_{max})$ **do**
  for i=1 to n $fireflies$ **do**
    for j=1 to n $fireflies$ **do**
      if $(I_j > I_i)$ **then**
        Move firefly i towards firefly j Eq. (3)
      end if
      Evaluate new solution and update light intensity Eq. (1)
    end for
    Rank fireflies and find the current best
  end for
  Post-process results and visualization
end while

$$I(r) = I_0 \cdot e^{-\gamma \cdot r^2} \tag{1}$$

where I_0 is the original light intensity at the location of $r = 0$, γ is the light absorption coefficient and r is the distance between two fireflies. The distance between any two fireflies i and j at x_i and x_j can be expressed as Cartesian distance $r_{ij} = ||x_i - x_j||$. As firefly attractiveness is proportional to the light intensity, we can define the attractiveness of a firefly using the following equation:

$$\beta(r) = \beta_0 e^{-\gamma \cdot r^2} \tag{2}$$

where β_0 is their attractiveness at $r = 0$. Firefly i that is attracted to another more attractive firefly j is determined by:

$$x_i = \beta_0 e^{-\gamma \cdot r^2} \cdot (x_j - x_i) + \alpha \cdot \varepsilon_i \tag{3}$$

which is randomized with the vector of random variable ε_i, being drawn from a Gaussian distribution, and step factor $\alpha \in [0, 1]$.

3. THE MULTI-POPULATION FIREFLY ALGORITHM

The multi-population firefly algorithm (MPFA) can be summarized in the pseudo-code as shown in Algorithm 2. MPFA will consider there to be an overall population P of Np fireflies (individuals) that is split into N sub-populations $P_1, P_2, ...P_N$. Each sub-population has Nsp individuals and the number of N sub-populations is calculated with the following equation:

$$N = \frac{Np}{Nsp} \tag{4}$$

For sub-populations to communicate with each other, the magnitude and frequency of that communication are necessary. These two parameters determine the amount of isolation and interaction between sub-populations. Periods of isolated evolution are referred to as *epoch*, with migration

Algorithm 2 Multi-Population Firefly Algorithm

Objective function $f(\mathbf{x})$, $\mathbf{x} = (x_1, ..., x_D)^T$
Calculate number of sub-populations (N)
for all $n \in N$ **do**
    Generate initial sub-population P_n of fireflies $\mathbf{x}_i$ (i=1,2,...,Np)
end for
Light intensity I_i at $\mathbf{x}_i$ is determined by $f(\mathbf{x}_i)$
Define light absorption coefficient γ
for e=1 to $\frac{G_{max}}{epoch}$ **do**
    for all $n \in N$ **do**
        for g=1 to *epoch* **do**
            for i=1 to n $fireflies$ **do**
                for j=1 to n $fireflies$ **do**
                    if $(I_j > I_i)$ **then**
                        Move firefly i towards firefly j
                    end if
                    Evaluate new solutions
                    Update light intensity
                end for
                Rank fireflies and find the current best
            end for
        end for
    end for
    Migrate fireflies
end for
Find the best firefly from all sub-populations
Post-process results and visualization

occurring at the end of each epoch except the last. The length of the epoch determines the frequency of interaction and is usually specified by a number of generations (*epoch*) that P_n evolves in isolation. During the epoch, each sub-population executes a sequential FA for *epoch* independently. At the end of each epoch, individuals are migrated between sub-populations. There are many various migration strategies in multi-population models. The following two models are described and used in this paper: island and mainland-island model.

3.1 Island Model

The island Model Firefly Algorithm (MPFA-In, where n determines the number of sub-populations) consist of islands, where islands are referred to as sub-populations. When each sub-population is executed a sequential FA for *epoch* generations, individuals are migrated between sub-populations, as shown in Algorithm 3. The magnitude of the communication is defined, for instance, as $N_m = 25\%$. Then, N_m percent of migrants are chosen for each sub-population which were exchanged with other sub-populations, as shown in Figure 1. Let us assume two sub-populations P_1 and P_2 with $Nsp = 10$ and $N_m = 20\%$ are defined. Then, two individuals from sub-population P_1 are exchanged with two individuals in sub-population P_2. Thus, the sizes of the sub-populations remain the same. After the algorithm reaches the termination criteria, the best individual is taken from all sub-populations.

3.2 Mainland-Island Model

The Mainland-Island Model Firefly Algorithm (MPFA-Mn, where n determines the number of sub-populations) consist of mainland and islands, where mainland and islands are referred to as sub-populations. When each sub-population has executed a sequential FA for G_i generations, individuals are migrated from sub-populations $P_2, ..., P_N$ to sub-population P_1, as shown in Algorithm 4. Let us assume two sub-populations P_1 and P_2 with $Nsp = 10$ and $N_m = 20\%$ are defined. Then, two individuals per sub-population $P2, ..., P_N$ are moved to sub-population P_1, as shown in Figure 2. At the end of migration, sub-population P_1 is sorted according to the fitness values of migrated individuals. In order to keep the size of sub-population P_1 the same, the top Nsp individuals are retained, while the others are discarded. After the algorithm has reached the terminating criteria, the best individual was taken from the sub-population P_1 (mainland).

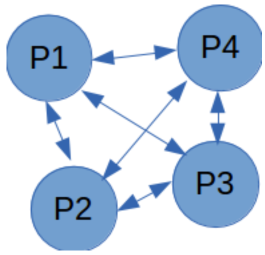


Figure 1: Island Model

Algorithm 3 Multi-Population Firefly Algorithm with Island Model - Migration

Get the number of migrants N_m to migrate
for $i = 1$ to N **do**
    Choose N_m individuals from P_i that are mutually different and save them to the matrix M
end for
Mutually exchange individuals that are defined in matrix M

Algorithm 4 Multi-Population Firefly Algorithm with Mainland-Island Model - Migration

Get the number of migrants N_m to migrate
for $i = 2$ to N **do**
    Choose N_m individuals from P_i that are mutually different and save them to the matrix M
end for
Copy chosen individuals from sub-populations $P_2, ..., P_N$ to sub-population P_1
Sort individuals in P_1 by light intensity
Keep top Nsp individuals, remove others, so the size of the sub-population P_1 remains the same

4. EXPERIMENTS AND RESULTS

The goal of this experimental work was to show that MPFA can outperform the results of the original FA algorithm. In our experiments, the results of the original FA were compared with the results of the following MPFA: MPFA-I2 and MPFA-I4 (i.e., MPFA with island model using two or four sub-populations), and MPFA-M2 and MPFA-M4 (i.e., MPFA with mainland-island model using two or four sub-populations). Additionally, the following three population models were used during tests, i.e., small with an original

Table 1: **Comparison between FA algorithms for population model Np=100 and D=10**

Func.	FA	MPFA-I2	MPFA-I4	MPFA-M2	MPFA-M4
1	1.0243e+06 ± 2.0147e+06	7.6582e+05 ± 3.4941e+06	6.4129+e05 ± 2.3202e+06	7.6582e+05 ± 3.4929e+06	**5.0199e+05** ± 9.3012e+05
2	1.3862e+04 ± 8.2668e+03	6.1151e+03 ± 4.8489e+03	**5.4070+e03** ± 4.0991e+03	6.1151e+03 ± 4.8489e+03	6.5121e+03 ± 8.0172e+03
3	2.0877e+04 ± 1.9394e+04	2.2167e+04 ± 2.0586e+04	**1.6543e+04** ± 1.3981e+04	2.2167e+04 ± 2.0586e+04	2.3253e+04 ± 2.6483e+04
4	7.7409e+00 ± 1.6024e+01	6.9890e+00 ± 1.1157e+01	7.0024e+00 ± 2.9768e+01	7.1320e+00 ± 1.0983e+01	**6.8717e+00** ± 8.8252e+00
5	2.0107e+01 ± 0.0704e+00	2.0059e+01 ± 0.0405e+00	**2.0035e+01** ± 0.0260e+00	2.0059e+01 ± 0.0405e+00	2.0044e+01 ± 0.0380e+00
6	8.4299e+00 ± 4.0688e+00	8.9432e+00 ± 2.5879e+00	**8.3829e+00** ± 2.6099e+00	8.9432e+00 ± 2.5879e+00	9.6953e+00 ± 3.2605e+00
7	5.4650e+00 ± 6.2455e+00	9.9181e+00 ± 1.0126e+01	5.8841e+00 ± 4.6803e+00	9.9181e+00 ± 1.0126e+01	**5.3097e+00** ± 5.9105e+00
8	**1.6925e+01** ± 1.5711e+01	2.3883e+01 ± 2.3057e+01	2.1892e+01 ± 1.4542e+01	2.3883e+01 ± 2.3057e+01	3.0847e+01 ± 2.5583e+01
9	**1.6924e+01** ± 1.2980e+01	1.8907e+01 ± 2.0379e+01	1.9903e+01 ± 1.6285e+01	1.8907e+01 ± 2.0379e+01	3.0847e+01 ± 2.7831e+01
10	1.1733e+03 ± 9.0738e+02	1.3110e+03 ± 1.1059e+03	**1.0351e+03** ± 8.9937e+02	1.3110e+03 ± 1.1157e+03	1.2901e+03 ± 1.4476e+03
11	1.1434e+03 ± 9.9968e+02	1.2193e+03 ± 1.0618e+03	**9.1012e+02** ± 8.3705e+02	1.2193e+03 ± 1.0618e+03	1.3165e+03 ± 1.1339e+03
12	0.4707e+00 ± 0.9716e+00	0.3272e+00 ± 1.0509e+00	**0.1942e+00** ± 0.4033e+00	0.3272e+00 ± 1.0509e+00	0.4343e+00 ± 1.7646e+00
13	0.3973e+00 ± 0.3133e+00	0.3949e+00 ± 0.3414e+00	**0.3137e+00** ± 0.2133e+00	0.3949e+00 ± 0.3414e+00	0.3578e+00 ± 0.3217e+00
14	0.3618e+00 ± 0.2481e+00	0.3578e+00 ± 0.2386e+00	**0.3260e+00** ± 0.1413e+00	0.3578e+00 ± 0.2386e+00	0.3378e+00 ± 0.2627e+00
15	**1.3559e+01** ± 1.5484e+01	1.7627e+01 ± 1.5247e+01	2.1337e+01 ± 1.8450e+01	1.7627e+01 ± 1.5247e+01	2.8024e+01 ± 2.6350e+01
16	3.7235e+00 ± 0.5893e+00	3.8952e+00 ± 0.7525e+00	**3.7126e+00** ± 0.7731e+00	3.8952e+00 ± 0.7525e+00	4.0390e+00 ± 0.7963e+00

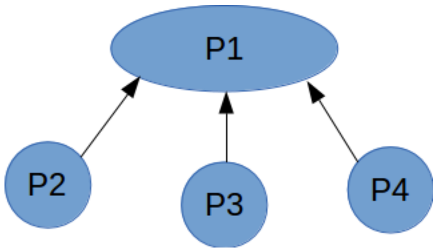


Figure 2: Mainland-Island Model

population size of $Np = 100$, medium with $Np = 200$ and large with $Np = 400$. The original population size was divided between sub-population according to Eq. (4). The same number of generations $G_{max} = 1,000$ was used for each sub-population for each algorithm.

The FA algorithms used the following parameter settings. The maximum number of evaluations was set as $MAX\_FEs = 10,000 \cdot D$. The randomized factor was fixed at $\alpha = 0.5$, the lights absorption at $\gamma = 1$, and the attractiveness at the beginning $\beta_0 = 1$. In each generation the randomized factor α was updated by the following equation: $\alpha = \alpha \cdot (1 - \delta)$, where $\delta = 1.0 - (\frac{10^4}{0.9})^{\frac{1}{G}}$ [5]. For the MPFA algorithms, some additional parameters were used, like the number of epochs as $epoch = 100$, and migration probability $N_m = 25\%$. Tests were conducted on all three population models using five FA algorithms, i.e., FA, MPFA-I2, MPFA-I4, MPFA-M2, and MPFA-M4. In summary, 15 tests were performed, in which 51 independent runs were performed.

All algorithms were tested on the 16 single-objective uni-modal and simple multi-modal CEC 2014 benchmark functions [7]. For uni-modal functions the convexity guarantees that the final optimal solution is also the global optimum. The global maximum was measured according to an error value ER. The error value for each function is calculated by subtracting the value of global optima from the obtained value according to the following equation:

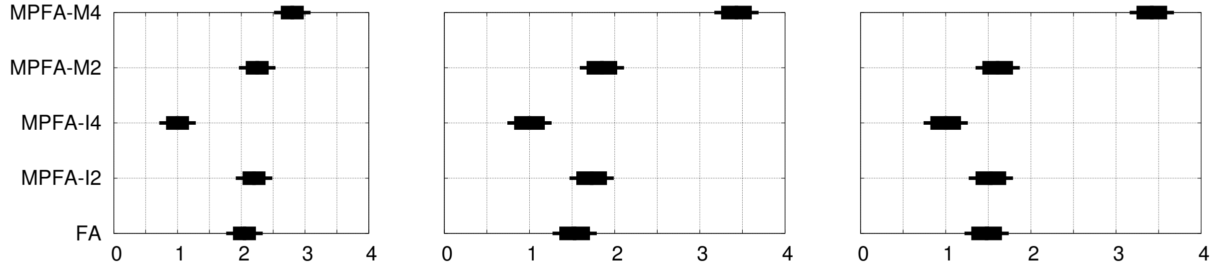
$$ER_i = f_i(\mathbf{x}) - f_i(x^*), \text{ [7]} \tag{5}$$

where i is the function number, $f_i(x)$ is the obtained value, and $f_i(x*) = 100 \cdot i$ is the value of global optima for i-th function. Note that error values smaller than 10^{-8} were taken as zero. In order to limit a search space, each problem variable can capture the value from the range $x_i \in [-100, 100]^D$, where values -100 and 100 represent its upper and lower bounds. The dimensionality of all problems was limited to $D = 10$.

The results of the mentioned FA algorithms are illustrated in Table 1. The results for all population models (i.e., small, medium, large) were obtained. Small population model ($Np = 100$) gave the best results, and due to the paper's length limitation only these results are presented in this table. In line with this, the original FA algorithm is compared with the MPFA-I2 and MPFA-M2 using two sub-populations of size $Nsp = 50$, and MPFA-I4 and MPFA-M4 using four sub-populations of size $Nsp = 25$. The table presents mean and standard deviation over 51 independent runs for each algorithm. The results in Table 1 show that MPFA-I2 as well as MPFA-M2 outperformed the original FA on 7 out of 16 test functions, MPFA-M4 on 8 out of 16 test functions and MPFA-I4 on 12 out of 16 test functions. The best results were obtained with MPFA-I4 which outperformed the other MPFAs and the original FA on 10 out of 16 functions.

In order to evaluate the quality of the results statistically, Friedman tests [6] were conducted that compare the average ranks of the compared algorithms. Thus, a null-hypothesis is placed that states: two algorithms are equivalent and therefore, their ranks should be equal. When the null-hypothesis is rejected, the Bonferroni-Dunn test [2] is performed. In this test, the critical difference is calculated between the average ranks of those two algorithms. If the statistical difference is higher than the critical difference, the algorithms are significantly different.

Three Friedman tests were performed regarding data obtained by optimizing 16 functions of three different population sizes according to five measures. As a result, each algorithm during the tests (also the classifier) was compared with regard to the 48 functions x 5 measurements this means, 240 different variables. The tests were conducted at a significance level of 0.05. The results of the Friedman non-parametric test can be seen in Figure 3 that is divided into three diagrams. Each diagram shows the ranks and confidence intervals (critical differences) for the algorithms under



(a) *Small* ($Np = 100$) (b) *Medium* ($Np = 200$) (c) *Large* ($Np = 400$)

Figure 3: Results of the Friedman non-parametric test

consideration with regard to the dimensions of the functions. Note that the significant difference between the two algorithms is observed if their confidence intervals denoted as thickened lines in Fig. 3 do not overlap.

As can be seen from Fig. 3, the MPFA-I4 outperformed the results of the original FA as well as the other algorithms using all three observed population size model significantly. The MPFA-M4 achieved the results that are significantly worse than the results of the other FA algorithms. The performances of the other three algorithms were comparable with each other.

5. CONCLUSION

In this paper, we proposed MPFA using two multi-population models, i.e., Island and Mainland-Island Models. The proposed MPFAs were compared with the original FA algorithm using three different population size models (i.e., small, medium, large) by solving the CEC-14 benchmark function suite. Based on the obtained results, we can see that the most promising results were obtained by the MPFA-I4. This fact encourage us to continue with the experiments of multi-population FA in the future.

The future work could be especially focused on the migration probability and dimension of the problem. Current migration probability was fixed for all multi-population models, but the migration probability can be modified or even adapted during the algorithm run. On the other hand, all the performed tests were done on small dimensions of the problem. Thus, the algorithm with few number of evaluations and larger population sizes did not reach the migration phase at all. With larger dimensions, the number of evaluations would be increased and the multi-population strategies could perform even better.

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