Večkriterijska optimizacija: eksperimentalni rezultati algoritmov *MOjDE* in *DEMO*

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Multiobjective optimization: Experimental Results of *MOjDE* and *DEMO* Algorithms

This paper is an extension of the paper [1]. Here we presents detailed experimental results obtained by our self-adaptive differential evolution algorithm, called $MOjDE$, which is an extension of $DEMO$ [4, 5] algorithm with self-adaptive control parameters, first proposed by J. Brest et al. [2].

1 Uvod

V članku prikazujemo rezultate poskusov za [1] na testnih funkcijah CEC'2007. Ti so prikazani v tabelah 1–6, kjer vrednost v tabeli predstavlja razliko med vrednostjo dobljeno z algoritmom $MOjDE$ [1, 2] in vrednostjo dobljeno z algoritmom $DEMO$ [4, 5]. Predstavitev z razliko nam omogoča lažjo primerjavo obeh algoritmov, saj v primeru pozitivne vrednosti to pomeni, da je algoritem $MOjDE$ boljši v primerjavi z algoritmom $DEMO$. Rezultati v tabelah so prikazani pri treh različnih številih evaluacij funkcij (FES): 5.000, 50.000 in 500.000 ($5e+5$), kjer zadnja vrednost predstavlja tudi najpomembnejši rezultat. Vseh zagonov za posamezno funkcijo je bilo 25. Velikost populacije je bila 100, 150 in 800 [3]. V tabeli vidimo najboljši rezultat (Best), mediano in najslabši rezultat ter povprečno vrednost in standardni odklon.

2 Statistična primerjava eksperimentov

Pri povprečnih vrednostih pri indikatorju R vidimo, da je algoritem $MOjDE$ pri $5e + 5$ FES 14-krat boljši in 5-krat slabši v primerjavi z $DEMO$. Če pa opravimo primerjavo s t-testom (pri 99,9% zanesljivosti), ugotovimo, da je $MOjDE$ v poprečju 13-krat (signifikantno) boljši, 4-krat slabši (S_ZDT2, R_ZDT4, S_ZDT6 in S_ZDTZ2 pri $M = 5$) in 2-krat t-test ne kaže signifikantne razlike. Pri povprečnih vrednostih pri indikatorju H vidimo, da algoritem $MOjDE$ pri $5e + 5$ FES pravtako 14-krat boljši in 5-krat slabši v primerjavi z $DEMO$. Če pa opravimo primerjavo s t-testom (pri 99,9% zanesljivosti), ugotovimo, da je $MOjDE$ v poprečju 13-krat (signifikantno) boljši, 4-krat slabši (S_ZDT2, R_ZDT4, S_ZDT6 in S_ZDTZ2 pri $M = 5$) in 2-krat t-test ne kaže signifikantne razlike. Na sliki 1 je grafično prikazana tudi bližina optimalnim rešitvam za algoritem $MOjDE$.

3 Zaključek

V prispevku smo statistično signifikantno pokazali, da je algoritem $MOjDE$ boljši kot $DEMO$ na določenih testnih funkcijah CEC'2007. Kot vidimo, je statistično signifikantna izboljšava vidna na večini uporabljenih testnih funkcijah. Do podobnih ugotovitev pridemo, če opravimo primerjavo najboljših (Best v tabelah 1–6) rezultatov večkriterijske optimizacije.

Literatura

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Tabela 1: Rezultati indikatorja R ($MOjDE - DEMO$) na testnih funkcijah 1–7.

FES		1. OKA2	2. SYMPART	3. S_ZDT1	4. S_ZDT2	5. S_ZDT4	6. R_ZDT4	7. S_ZDT6
$5e+3$	Best	0.0000e+00	-1.2126e-02	-2.8753e-02	-3.1830e-02	-2.0340e-03	-3.2850e-03	-1.1560e-02
	Median	6.3839e-04	-8.9560e-03	-2.8680e-02	-2.5153e-02	-5.7430e-03	7.4800e-04	-8.2700e-03
	Worst	1.0872e-03	-1.3827e-02	-1.3948e-02	-2.3340e-02	-7.5850e-03	-6.7930e-03	-4.5500e-03
	Mean	*2.7546e-04*‡	**-9.4520e-03**†	**-2.7496e-02**†	**-2.6850e-02**†	**-4.4840e-03**†	2.5600e-04	**-8.2400e-03**†
	Std	5.0059e-04	-8.7700e-05	2.9351e-03	9.0610e-04	-5.5550e-04	-7.7600e-05	2.1515e-03
$5e+4$	Best	0.0000e+00	-7.7090e-06	-2.0767e-04	-4.8480e-04	-2.4651e-02	-1.8655e-03	-2.8215e-02
	Median	-4.2960e-05	-5.2360e-06	-2.5016e-04	3.9140e-02	-2.5571e-02	-2.5940e-03	2.7900e-04
	Worst	-2.2382e-03	1.7136e-05	4.9818e-04	1.2190e-03	-2.4851e-02	-3.3543e-03	2.3928e-02
	Mean	**-1.8118e-04**†	**-3.2750e-06**†	**-1.7308e-04**†	*9.1070e-03*‡	**-2.5738e-02**†	**-2.4415e-03**†	-2.5530e-03
	Std	-4.2259e-04	3.9093e-06	1.7599e-04	-1.1000e-03	-5.4990e-04	-2.8291e-04	1.7770e-02
$5e+5$	Best	0.0000e+00	-1.8015e-06	-1.0798e-07	0.0000e+00	-2.4653e-03	-1.1610e-04	0.0000e+00
	Median	8.2980e-05	-2.5904e-06	-1.4496e-06	4.0053e-02	-3.6698e-03	5.1565e-04	0.0000e+00
	Worst	-4.0540e-05	-2.7899e-06	-1.2305e-05	4.2000e-05	-5.2097e-03	2.2226e-03	5.2671e-02
	Mean	1.4970e-05	**-2.5682e-06**†	**-1.8539e-06**†	*9.6150e-03*‡	**-3.6838e-03**†	*6.3149e-04*‡	*2.8515e-03*‡
	Std	-3.8840e-05	-1.7634e-07	-2.3841e-06	-1.2210e-03	-7.1145e-04	5.8445e-04	1.1026e-02

Tabela 2: Rezultati indikatorja R ($MOjDE - DEMO$) na testnih funkcijah 8–13 pri $M = 3$.

FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
$5e+3$	Best	-5.3028e-05	-5.1069e-04	3.8190e-05	4.1700e-04	1.0800e-03	-2.1040e-04
	Median	-8.0530e-05	-1.3407e-03	2.4710e-05	3.8900e-04	1.3200e-03	-4.6580e-04
	Worst	-1.1105e-04	-2.3156e-03	4.5240e-05	1.6800e-04	-1.3121e-03	7.8530e-04
	Mean	**-8.2090e-05**†	**-1.3380e-03**†	*2.7890e-05*‡	*3.4300e-04*‡	*1.0040e-03*‡	**-2.3790e-04**†
	Std	-1.8055e-05	-4.9209e-04	-3.6370e-06	-4.9040e-05	-4.6253e-04	7.0600e-05
$5e+4$	Best	4.6163e-06	-6.2530e-05	-5.1672e-05	-1.8640e-03	-9.1800e-04	-1.6460e-04
	Median	8.4790e-06	-8.5387e-04	-4.7664e-05	-1.5680e-03	-9.8100e-04	-2.8030e-04
	Worst	1.6554e-05	-6.2640e-04	-3.7205e-05	-1.6620e-03	-2.1470e-03	-1.9200e-04
	Mean	*7.0040e-06*‡	**-7.9841e-04**†	**-4.7950e-05**†	**-1.7640e-03**†	**-1.1830e-03**†	**-2.6500e-04**†
	Std	1.7970e-06	-2.2753e-04	3.4474e-06	2.0458e-04	-1.1439e-04	4.3130e-05
$5e+5$	Best	-6.3894e-06	-5.6360e-05	-1.1496e-05	-3.1745e-02	-8.7800e-04	-3.0810e-04
	Median	-4.3330e-06	-6.5920e-05	-1.5302e-05	-3.2012e-02	-8.2500e-04	-9.3900e-05
	Worst	2.0224e-05	6.9200e-05	-1.8617e-05	-2.5835e-02	-1.1310e-03	-1.6560e-04
	Mean	-2.8200e-07	**-2.1305e-04**†	**-1.5195e-05**†	**-3.0841e-02**†	**-8.4300e-04**†	**-2.1080e-04**†
	Std	4.0200e-06	-8.7420e-05	-1.8204e-06	1.6856e-03	-5.7500e-06	-1.9160e-05

Tabela 3: Rezultati indikatorja R ($MOjDE - DEMO$) na testnih funkcijah 8–13 pri $M = 5$.

FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
$5e+3$	Best	-3.4300e-06	-3.2150e-05	-6.8700e-06	1.5000e-04	-2.3590e-04	-2.5696e-03
	Median	9.0080e-05	-4.1994e-04	-8.8200e-06	1.0000e-04	9.1430e-04	-2.5423e-03
	Worst	7.1457e-04	-6.4117e-04	-6.4120e-05	9.3000e-05	-4.1100e-04	-3.6740e-03
	Mean	*1.4582e-04*‡	**-4.3424e-04**†	**-1.5810e-05**†	*1.0700e-04*‡	*6.9080e-04*‡	**-2.5105e-03**†
	Std	1.3899e-04	-1.5939e-04	-1.3524e-05	7.8800e-07	-3.8241e-04	-3.6930e-04
$5e+4$	Best	-1.8080e-06	-2.3589e-05	-2.6400e-07	1.5700e-04	6.4240e-04	-5.2000e-06
	Median	-6.9710e-06	-2.3472e-04	4.6900e-07	1.6400e-04	8.1910e-04	-2.4450e-04
	Worst	-1.5371e-05	-4.7879e-04	3.3510e-06	1.8500e-04	3.0290e-04	-3.2130e-04
	Mean	**-7.7630e-06**†	**-2.0500e-04**†	6.1400e-07	*1.7500e-04*‡	*6.6060e-04*‡	**-3.0250e-04**†
	Std	-2.4164e-06	-1.3982e-04	1.3248e-06	1.5030e-05	3.8540e-04	-6.2540e-05
$5e+5$	Best	4.3640e-07	-1.1624e-05	-4.0925e-06	-7.6300e-04	-4.3294e-03	-9.6040e-04
	Median	2.4038e-06	-2.3078e-05	-2.2825e-06	-6.3400e-04	-3.3114e-03	-2.9340e-04
	Worst	6.6200e-06	-2.6148e-04	-3.0939e-06	-1.5300e-04	-3.2264e-03	-2.5600e-04
	Mean	*2.8375e-06*‡	**-3.7360e-05**†	**-3.0186e-06**†	**-6.4100e-04**†	**-3.5630e-03**†	**-3.4310e-04**†
	Std	6.3600e-07	-4.9224e-05	5.2162e-07	1.3139e-04	2.0728e-04	9.5570e-05

Tabela 4: Rezultati indikatorja H ($MOjDE - DEMO$) on test functions 1–7.

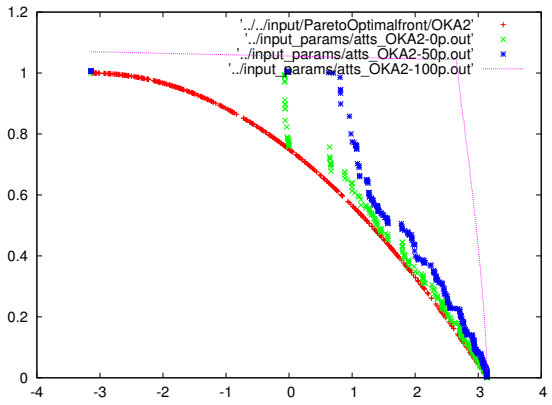
FES		1. OKA2	2. SYMPART	3. S_ZDT1	4. S_ZDT2	5. S_ZDT4	6. R_ZDT4	7. S_ZDT6
$5e+3$	Best	1.1630e-03	-3.4449e-02	-1.0462e-01	-8.8990e-02	-2.4000e-04	-4.2510e-03	-3.4930e-02
	Median	1.3650e-03	-2.4944e-02	-1.0623e-01	-8.7050e-02	-1.9340e-02	3.7760e-03	-2.5050e-02
	Worst	2.5710e-03	-3.7720e-02	-5.1460e-02	-7.9120e-02	-2.4570e-02	-2.3300e-02	-1.3140e-02
	Mean	*1.0060e-03*‡	**-2.6297e-02**†	**-9.9650e-02**†	**-9.1200e-02**†	**-1.5020e-02**†	1.2800e-03	**-2.4510e-02**†
	Std	1.2451e-03	6.5000e-05	1.0303e-02	3.3430e-03	-1.6140e-03	-4.4500e-04	5.8331e-03
$5e+4$	Best	-4.4660e-03	-2.2860e-05	-1.0724e-03	-1.2995e-03	-7.2292e-02	-5.0227e-03	-6.5834e-02
	Median	-5.1870e-03	-1.6020e-05	-1.1787e-03	4.5359e-02	-7.5265e-02	-7.0666e-03	-1.1600e-04
	Worst	-1.4400e-03	5.0130e-05	1.7311e-03	3.6800e-03	-7.3575e-02	-9.9980e-03	5.6813e-02
	Mean	**-5.0920e-03**†	**-9.8280e-06**†	**-8.5209e-04**†	*1.0095e-02*‡	**-7.5794e-02**†	**-7.0770e-03**†	**-6.6880e-03**†
	Std	1.1630e-03	1.1403e-05	6.8161e-04	-1.2320e-03	-1.7818e-03	-9.1560e-04	4.1712e-02
$5e+5$	Best	-7.5272e-03	-6.1565e-06	2.2300e-06	2.7500e-06	-7.7217e-03	-3.1899e-04	7.2000e-07
	Median	-7.4021e-03	-7.8399e-06	-7.1000e-07	4.7621e-02	-1.0948e-02	1.4879e-03	5.2000e-07
	Worst	2.6320e-03	-8.0360e-06	-2.7290e-05	1.2700e-04	-1.5396e-02	6.5839e-03	1.2059e-01
	Mean	**-6.8852e-03**†	**-7.7318e-06**†	-1.2000e-06	*1.1437e-02*‡	**-1.1146e-02**†	*1.8427e-03*‡	*6.2687e-03*‡
	Std	2.3698e-03	-4.3580e-07	-4.7232e-06	-1.4510e-03	-1.9325e-03	1.7334e-03	2.4887e-02

Tabela 5: Rezultati indikatorja H ($MOjDE - DEMO$) on test functions 8–13 pri $M = 3$.

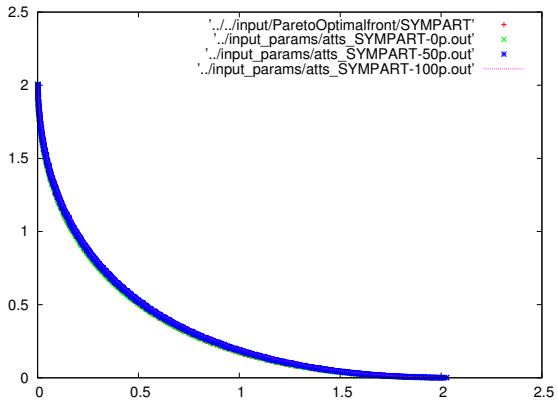
FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
$5e+3$	Best	-1.3457e-03	-6.7980e-03	-1.1392e-03	1.3900e-03	1.6400e-03	-1.6130e-03
	Median	-1.5223e-03	-6.6920e-03	1.3496e-03	1.6100e-03	5.0300e-03	-6.0000e-04
	Worst	-2.1695e-03	-1.1887e-02	3.7590e-03	8.6000e-04	-2.0650e-03	-3.0200e-04
	Mean	**-1.5319e-03**†	**-7.9640e-03**†	*9.6400e-04*‡	*1.4100e-03*‡	*4.8980e-03*‡	**-1.9460e-03**†
	Std	-1.2391e-04	-2.3820e-03	7.6650e-04	-1.8958e-04	-1.0643e-03	-4.2600e-04
$5e+4$	Best	1.3271e-05	-2.2993e-03	-2.1509e-04	-9.4500e-03	-4.7200e-03	-2.0730e-03
	Median	2.7897e-05	-5.9840e-03	-3.0073e-04	-8.0000e-03	-5.2400e-03	-1.6340e-03
	Worst	5.0630e-05	-2.9660e-03	-3.0146e-04	-8.0900e-03	-1.0370e-02	-2.1750e-03
	Mean	*1.6758e-05*‡	**-5.4270e-03**†	**-2.8428e-04**†	**-8.6500e-03**†	**-6.1000e-03**†	**-1.7360e-03**†
	Std	5.6060e-06	-6.2190e-04	-2.8164e-05	1.0581e-03	-5.7590e-04	-2.0000e-04
$5e+5$	Best	1.3434e-05	-6.4450e-04	-8.7981e-07	-1.6748e-01	-4.1200e-03	-1.5530e-03
	Median	3.8180e-06	-1.1760e-03	-2.1918e-06	-1.6728e-01	-3.9100e-03	-1.4060e-03
	Worst	-8.2160e-05	-8.0500e-04	-3.5916e-06	-1.3362e-01	-4.0500e-03	-2.6100e-04
	Mean	1.9700e-06	**-1.8190e-03**†	**-2.1996e-06**†	**-1.6128e-01**†	**-3.6900e-03**†	**-1.1600e-03**†
	Std	-1.0086e-05	-4.3820e-04	-8.8248e-07	9.6909e-03	3.5690e-05	-9.4000e-06

Tabela 6: Rezultati indikatorja H ($MOjDE - DEMO$) on test functions 8–13 pri $M = 3$.

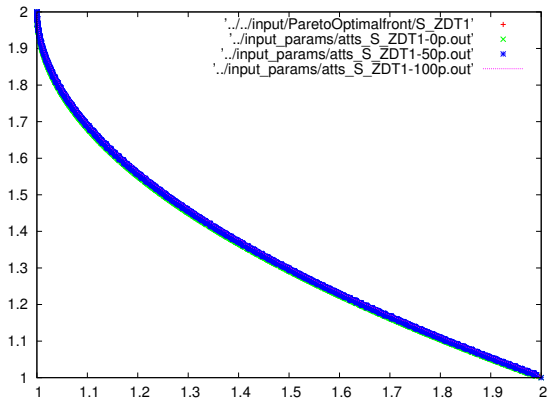
FES		8. S_DTLZ2	9. R_DTLZ2	10. S_DTLZ3	11. WFG1	12. WFG8	13. WFG9
$5e+3$	Best	3.1470e-03	-2.8150e-03	8.3300e-05	1.0400e-03	-1.2820e-03	-1.2427e-02
	Median	7.8190e-03	-7.6090e-03	-4.7000e-04	1.1300e-03	-1.3559e-02	-1.5900e-02
	Worst	1.3298e-02	-1.0863e-02	-8.9200e-04	7.2000e-04	-1.0615e-02	-2.8390e-02
	Mean	*8.1690e-03*‡	**-7.6070e-03**†	**-3.3560e-04**†	*1.1000e-03*‡	**-1.0359e-02**†	**-1.9930e-02**†
	Std	2.9399e-03	-2.0770e-03	-3.4250e-04	6.9630e-05	-4.1315e-03	-3.5730e-03
$5e+4$	Best	-1.9029e-05	-2.0475e-03	-1.1981e-05	1.6100e-03	-2.8900e-03	-3.3200e-03
	Median	-2.9063e-05	-4.5694e-03	-1.5801e-05	1.5800e-03	1.7760e-02	-8.5500e-03
	Worst	-3.9793e-05	-7.8240e-03	-2.1998e-05	1.5900e-03	1.4280e-02	-1.8090e-02
	Mean	**-2.8730e-05**†	**-4.3448e-03**†	**-1.8081e-05**†	*1.6000e-03*‡	*1.5690e-02*‡	**-8.8590e-03**†
	Std	-4.9867e-06	-1.6657e-03	-6.2010e-07	-6.1700e-05	5.0476e-03	-3.9723e-03
$5e+5$	Best	7.3900e-09	-1.2052e-03	-2.4902e-07	-9.6500e-03	-4.4590e-02	-8.1000e-03
	Median	-8.9500e-08	-1.0010e-03	-6.5330e-07	-7.2300e-03	-4.7450e-02	-4.7400e-03
	Worst	1.9463e-06	-3.6959e-03	-1.3875e-06	-2.2800e-03	-4.8810e-02	-4.8100e-03
	Mean	*2.6920e-07*‡	**-1.0650e-03**†	**-6.6196e-07**†	**-6.5200e-03**†	**-4.6330e-02**†	**-4.7700e-03**†
	Std	6.3927e-07	-3.2217e-04	-2.7574e-07	1.1593e-03	1.6444e-03	6.4980e-04



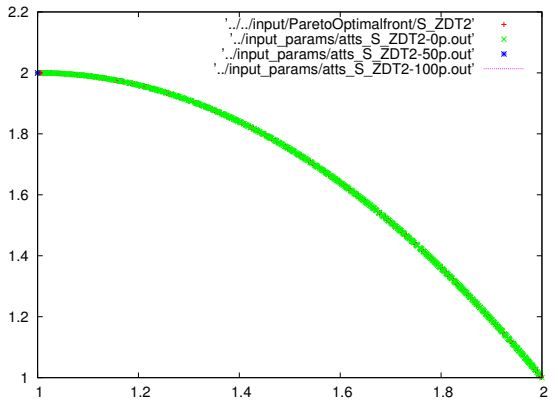
(a) MOP 1



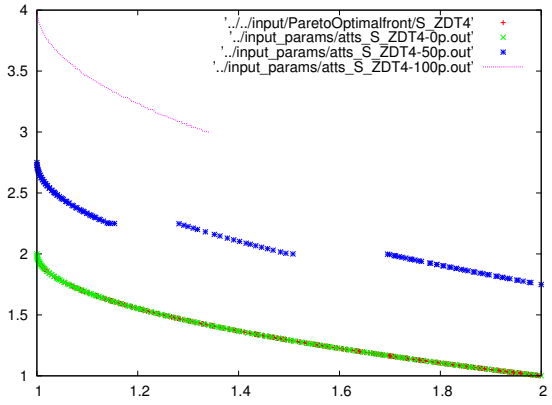
(b) MOP 2



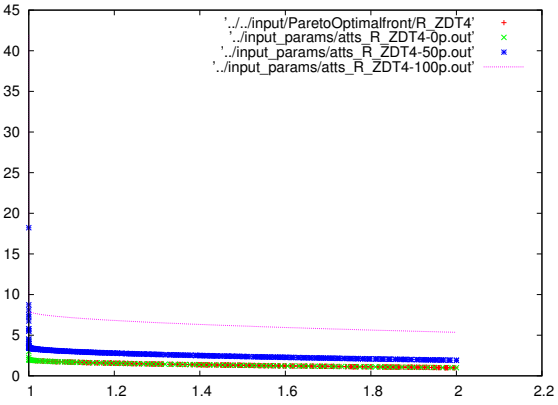
(c) MOP 3



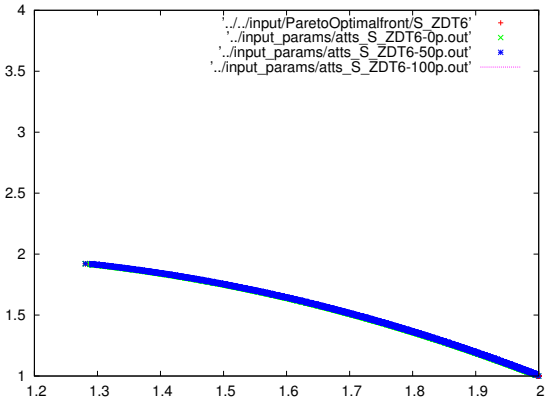
(d) MOP 4



(e) MOP 5



(f) MOP 6



(g) MOP 7

Slika 1: Empirično 0%, 50% in 100% dosegljiva ploskev vseh 25 zagonov na tesnih funkcijah 1–7 po $5e + 5$ evaluacijah večkriterijske funkcije.